Complete Search

Sometimes you need to check all the things….

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Objectives

- Describe four patterns of brute force;
- Describe the times when a brute force solution is necessary.
- Describe some techniques to optimize brute force algorithms.

What is it?

- You must traverse the entire problem space to get the answer.
- Sometimes you can prune the problem space.

8	6	7	5	3	0	9

```
max=a[0]; // why not just put 0 here?
for(int i=1; i<7; i++)
    if (a[i]>max) max=a[i];
```

When to Use It

- Tradeoffs
  - Bad: It's slow!
  - Good: It's simple! More likely to give correct solution.
- Three situations:
  - When you have no choice.
  - When the problem set is small.
  - To verify your real solution

Categories

- Code Pattern
  - Iterative
  - Recursive
- Traversal Pattern
  - Filtering
  - Generating

Speed

- Use bits instead of boolean arrays
- Use primitive types when appropriate:
  - int32 instead of int64
  - arrays instead of vector
  - character arrays instead of string
- Prefer iteration to recursion
- The STL algorithm include has `next_permutation`, which is very fast
- Declare large data structures in the global scope

The n-queens problem

	x		x	
x	x	x		
x	Q	x	x	x
x	x	x		
	x		x	

- If you don't know chess, you might want to learn the basic rules.
- Classic problem: place n queens on a $n \times n$ chessboard.
  - How many ways can you do it?

Two examples

Q				
			Q	
	Q			
				Q
		Q		

Q				
		Q		
				Q
	Q			
			Q	

How to write this?

- Attempt 1 : Massive nested `for` loops

```
vvi board(8,vi(8));  // get
count = 0;
for(i=0; i<7; ++i) {
  board[0][i] = 1; // place queen
  for(j=0; j<7; ++j)
    if (! collides(board,1,j)) {
      board[1][j] = 1;
      for(k=0; ...)  ;    // rest of program
      board[1][j] = 0;
    }
  board[0][i] = 0; // remove queen
```

- Final position; if no collisions increment `count`.
- What do you think of this code? 8^8 attempts. . . .

Improvements

Q				
		Q		
				Q
	Q			
			Q	

- We don't need 8^8 checks.
- Instead of modeling the chess board, model where the queens are placed.
  - This example is `{0,3,1,4,2}`.

Backtracking

- Example code from *Competive Programming 4*

```
void backtrack(int c) {
  if ((c == 8) && (row[b] == a)) { // a candidate solution
    printf("%2d %d", ++lineCounter, row[0]+1);
    for (int j = 1; j < 8; ++j)
      printf(" %d", row[j]+1);
    printf("\n");
    return; // optional statement
  }
  for (int r = 0; r < 8; ++r) { // try all possible row
    if ((c == b) && (r != a)) continue; // early pruning
    if (canPlace(r, c)) // can place a Queen here?
      row[c] = r, backtrack(c+1); // put here and recurse
  }
}
```

Checking Placement

- Don't walk the diagonals; use math!

```
bool canPlace(int r, int c) {
  for (int prev = 0; prev < c; ++prev) // check previo
    if ((row[prev] == r) ||
        (abs(row[prev]-r) == abs(prev-c)))
      return false; // infeasible
  return true;
}
```

Using bitmasks

- Keep three bit-vectors.
- Shifting the bits handles the diagonals.

```
// Place a queen in row $r$
rows |= (1 << r);
up |= (1 << r);
down |= (1 << r);

// Moving to the next column...

up <<= 1;
down >>= 1;
```